# Damage Surface Based on the Critical Crack Orientation

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Addessio and Johnson [1] proposed a constitutive model (ISOSCM) for the dynamic response of brittle materials (e.g., ceramics, high explosives, berylliums) based on the work of Dienes. They assumed that during the damage process the distribution of cracks remains random and isotropic, and that the crack probability density function is exponential in the crack size. The macroscopic crack strains and an isotropic damage surface are found by averaging the crack strains as well as the instability condition for a single crack over all crack orientations. The resulting isotropic damage surface involves only the mean crack size, the von Mises stress and pressure, and takes two different forms depending on the sign of the pressure. The damage evolution is given through the growth of mean crack size when the stress state is on or above the damage surface. The model is compatible with the incremental continuum formulation inherent to existing design computer codes and can be easily implemented into the codes. The model was applied to simulate damage in ceramics under impact conditions, and the predictions compared favorably with shock compression and release experiments.

Due to its numerical efficiency and mathematical simplicity and its origin in micromechanics, the model has been adopted by many researchers as the starting point for their models. Bennett et al. [2] extended ISOSCM to include the viscous effects of the binder materials to study the non-shock ignition of a plastic bonded explosive. Lee et al. [3] adopted the damage surface and the crack growth law developed in ISOSCM for modeling damage of fiber composites. There are still two issues in ISOSCM that warrant further improvement, namely, (1) a discontinuity in the damage surface as the pressure in the material changes sign, and (2) when the principal stresses have mixed signs, only the open cracks should contribute to the open crack strain. In ISOSCM, however, whether a crack would contribute to the open crack strain or not depends solely on the sign of the pressure in the material, not on its orientation (hence the sign of the traction). As discussed by Lewis and Schreyer [4], both issues could lead to thermodynamic inconsistency, which manifests into energy creation under certain cyclic load paths. The objectives of the current work are to develop an improved version of ISOSCM and to apply it to problems of damage and spall in beryllium under dynamic loading. Here, we present our new formulation for the damage surface which removes the discontinuity in the damage surface in ISOSCM.

When the applied stress is large enough, some cracks in the material can become unstable and grow in size. We assume here that the material accumulates additional damage if the crack with average size $\bar{c}$ along some orientation is unstable. The generalized Griffith instability criterion for a single crack is $F(\boldsymbol{\sigma},\boldsymbol{n},\bar{c}) = f(\boldsymbol{\sigma},\boldsymbol{n}) - \pi(2-\nu)G\gamma/(2\bar{c}(1-\nu)) = 0$, $\boldsymbol{\sigma}$ where is the remote stress tensor, $\boldsymbol{n}$ the unit normal of the crack, γ the surface energy of the material, and the elastic constants G and ν are the shear modulus and Poisson's ratio, respectively. The expression for the function $f(\boldsymbol{\sigma},\boldsymbol{n})$, which is proportional to the energy release rate, depends on whether the crack is open (the normal component of traction is tensile) or closed (the normal component is compressive and controls the interfacial friction, see [5] for details). The crack instability surface $F(\boldsymbol{\sigma},\boldsymbol{n},\bar{c}) = 0$ in the $\sigma_n - S_n$ plane, along with the Mohr circles, which conveniently relate the normal and shear components of traction on a surface to the principal stresses, is given in Fig. 1. It is interesting to note that the instability surface for closed cracks coincides with the Mohr-Coulomb failure envelope. The instability condition for a penny-shaped crack may provide a justification for the Mohr-Coulomb criterion for brittle materials and the means to relate the cohesion constant to the defects (crack radius) in the materials.

The damage surface for the material, $\bar{F}(\sigma,\bar{c}) = 0$, is found by applying the instability condition to the critical (most unstable) crack orientation. For a given crack size and stress state, the critical crack orientation, $\boldsymbol{n}^*$ maximizes the function

$f(\sigma, n)$, and under the assumption of a random, isotropic distribution of cracks, the crack along that direction will first become unstable when the applied stress is high enough.

We have recently found [5] the critical crack orientation and determined the load that is needed for the crack along that orientation to become unstable. Figure 2 is a comparison of the new damage surface based on the instability condition of the critical crack orientation and that based on averaging the crack instability condition over all crack orientations [1] for the same set of material constants. The state of stress is taken to be triaxial with uniform lateral confinement. The Poisson's ratio and friction coefficient are $\nu = 0.25$ and $\mu = 0.2$, and the pressure and von Mises stress are normalized with respect to $P_t = \sqrt{\pi G \gamma / (\bar{c}(1-\nu))}$, the hydrostatic tensile stress the material can sustain before it accumulates additional damage. It may be observed that the surface based on the critical crack orientation is continuous, whereas the surface based on averaging has a jump in the von Mises stress as the pressure changes sign. It is also seen that both surfaces predict the same value when material is under hydrostatic tension, as expected, since in this case all orientations are equally critical and the two approaches are identical. Both approaches also predict no damage evolution (crack growth) under hydrostatic compression, consistent with crack mechanics.

For a non-isotropic stress state ($\tau > 0$), the current approach predicts less shear a material can sustain before additional damage accrues than the previous approach. This is consistent with the assumptions in the current approach that damage grows when the critical orientation becomes unstable while the previous approach assumes that damage can grow only when the "averaged" orientation becomes unstable.

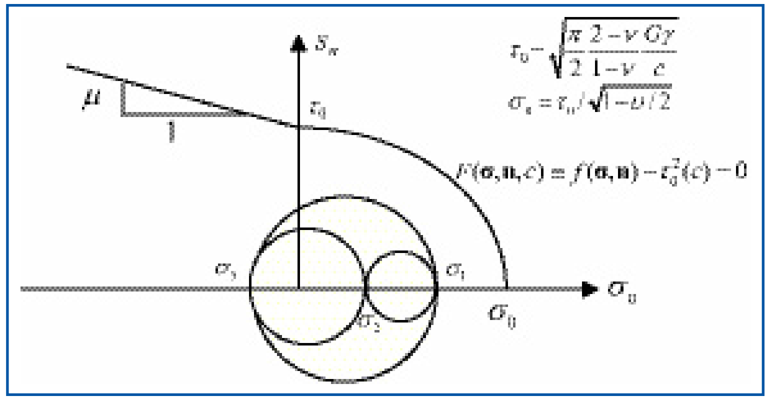


***Figure 1—***
***The instability surface of a penny-shaped crack under combined shear ($s_n$) and normal ($\sigma_n$) stresses with interfacial friction.***

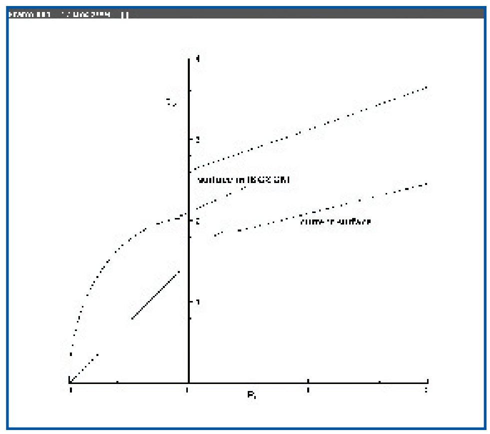

***Figure 2—***
***Comparison of the new damage surface and that developed previously [1].***

[1] F.L. Addessio and J.N Johnson, "A Constitutive Model for the Dynamic Response of Brittle Materials," *J. Appl. Phys.* **67**, 3275 (1990).
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[3] H.K. Lee, S. Simunovic, and D.K. Shin, *Comput. Mat. Sci.* **29**, 459 (2004).
[4] M.W. Lewis and H.L Schreyer, "A Thermodynamically Consistent Description of Dynamic Continuum Damage," in *High Pressure Shock Compression of Solids*, L. Davison, D.E. Grady, and M. Shahinpoor, Eds. (New York: Springer-Verlag 1996) Vol. II, p. 452.
[5] Q.H. Zuo and J.K. Dienes, "On the Stability of Penny-Shaped Cracks with Friction: The Five Types of Brittle Behavior," *Int. J. Solids Struct.* **42**, 5–6 1309–1326 (March 2005).